Longitude Positioning and Orbit Control of the 24-Hour Equatorial Satellite

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This paper presents a systems analysis approach to the problems involved in obtaining and maintaining a 24-hr equatorial satellite Consideration is given to the problems involved in orbit ascent, initial longitude positioning, and subsequent control of the satellite drift away from the desired stationary longitude A complete orbit control system, including ground based tracking and command stations, computations and control center, and satellite subsystems, which will provide a satisfactory stationary orbit is synthesized The capability of the control system to satisfactorily position and control a 24-hr equatorial satellite was then tested by a detailed digital computer simulation of the complete orbit control system The results of the simulations indicate that the synthesized system can provide a satisfactory stationary orbit in the presence of likely system errors

I Introduction

ANALYSIS of the 24-hr, equatorial satellite mission requires consideration of the methods to be used and the problems involved in 1) ascent to altitude (19,327 naut miles) and injection, 2) longitude positioning over the desired stationary longitude, and 3) station keeping to maintain the satellite about the desired stationary longitude In order to analyze the problems associated with each of these phases in obtaining and maintaining the desired longitude, a systems analysis approach has been used Using this approach, an orbit control system has been synthesized which has as its primary function the attainment and maintenance of a desired stationary longitude In synthesizing the orbit control system, consideration has been given to the methods of longitude positioning and station keeping to be used, satellite propulsion and attitude control requirements, initial and final orbit accuracy requirements, and ground tracking and command station configurations

The complete orbit control system can be considered to consist of three major subsystems 1) tracking and command stations, 2) control and computations center, and 3) satellite and associated subsystems

Figure 1 presents a block diagram of the conceptual orbit control system For purpose of orbit control, the control and computations center has the primary functions of 1) tracking data conversion and editing, 2) orbit determination, 3) ephemeris generation, 4) orbital computation, 5) orbit control and tracking decisions, 6) command computations, and 7) issuance of appropriate tracking and command instruction, to ground stations

The tracking and command ground stations will forward radar tracking data to the control center in real time It has been assumed that the ground tracking stations will measure radar range rate, azimuth angle, and elevation angle Using these measurements, the control center will do an orbit determination and determine whether or not a correction should be made If a correction is to be made, the control center will compute the required velocity correction and associated command quantities and forward them to the appropriate ground command station for transmittal to the satellite At the appointed time, the command station will transmit the required commands to the satellite, verify receipt of commands, and send the execute signal

Upon receipt of the execute command, the satellite's programmer will engage the attitude control system to align the satellite in the proper orientation for making a correction, initiate thrusting of the selected propulsion unit, and provide a thrust cutoff signal after the commanded thrusting time has elapsed

The complete procedure is then repeated for as many corrections as the control center decides are necessary Thus, the orbit control system functions in a closed-loop fashion, utilizing control cycles consisting of tracking, orbit determination, guidance or orbit control decisions, command transmission, and satellite subsystem command execution

In order to test realistically the capability of the synthesized orbit control system to position and control a 24-hr equatorial satellite, the complete system was simulated on an IBM digital computer A series of Monte Carlo runs, including noise errors on the appropriate subsystems, was then made The results of these runs indicate that the system can provide a satisfactory stationary orbit

II Orbit Control System Synthesis

A Method of Ascent

In general, the ascent to altitude and injection, and longitude positioning phases are not mutually exclusive, in as much as the booster ascent trajectory may incorporate suitable dwell maneuvers so as to achieve the desired altitude and longitude simultaneously The various methods of ascent are analyzed in Ref 1 Their principal characteristic of concern here is the incorporation of a degree of freedom, which allows achievement of the desired longitude upon arrival at altitude

For many reasons, an alternate technique of achieving the desired station (one that separates ascent and longitude positioning) is more desirable and has been selected for use in the over-all orbit control system This is the so-called "walking-orbit" or "biased-injection" technique, which features a fixed ascent to altitude; injection into an equatorial orbit having a period less than 24 hr; and subsequent control of the vehicle's apparent eastward drift in longitude until the desired station is achieved Inertially, the walking-orbit is actually a high altitude, elliptic parking orbit having apogee at the 24-hr alt Control of the apparent longitude

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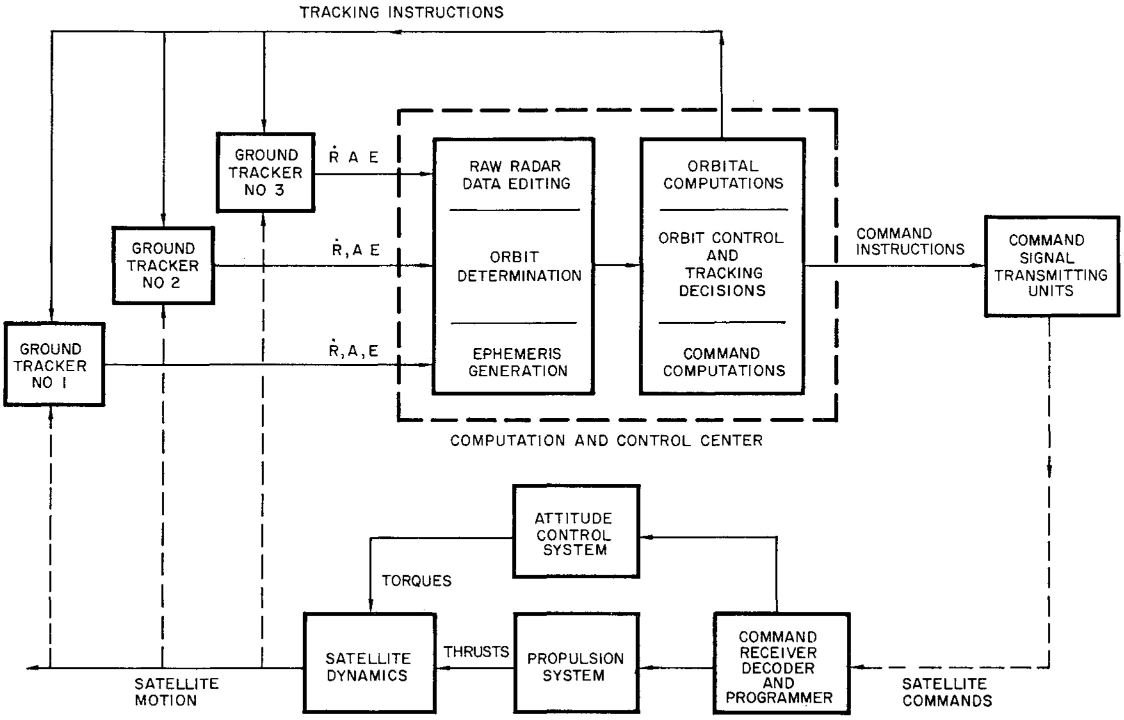

Fig 1 Block diagram of conceptual orbit control system

drift per orbit revolution is achieved through orbit period monitoring, which, in turn, is realized through apogee speed control

The apparent longitude drift per orbit revolution (briefly, drift rate) is given by

$$\alpha = \omega (T_s - T) \tag{1}$$

where α is the (eastward) longitude drift rate, ω the sidereal rate of earth's rotation, T_s the sidereal period of earth's rotation, and T the satellite's orbital period

The relation between α, T, and apogee speed is shown in Fig 2 in terms of a velocity deficit from circular velocity

It should be noted that eccentricity control is implicit in period control if all impulses are scheduled at apogee and are directed circumferentially Thus, by adjusting the period of the walking-orbit so that the desired longitude falls under an apogee point, final adjustment of the period nominally results in a circular 24-hr orbit with the desired stationary longitude

B System Accuracy Requirements

The requirements that the system must satisfy are determined by the orbit specifications and ground station tracking accuracies The orbit specifications are 1) desired longitude = λ , 2) allowable steady longitude error < D_s (i e , the number of degrees of longitude away from λ_s which the satellite will be allowed to drift over the system lifetime), 3) diurnal longitude variation < D_e, 4) maximum latitude error = i_s, and 5) system operating lifespan = N_L The diurnal longitude variation and the maximum latitude error convert into e and i requirements For this system it has been assumed that these requirements will be of such a magnitude that neither e nor i need be explicitly controlled [18] Thus, the value of steady longitude error and system lifetime will determine how accurately the system must be controlled

Adequate achievement of the stationary property requires extreme precision For example, the secular drift in longitude caused by an orbital period error of 1 sec is approximately 1 5°/yr Thus, if it is required that the satellite remain within 5° of its desired longitude over an operating span of 5 yr, the orbit period must be controlled to within 0 67 sec At the 24-hr alt, this is equivalent to a speed accuracy of 0 027 fps From these considerations it is apparent that closed-loop control employing ground tracking of the satellite is required

Since a secular longitude drift of 1°/yr does not appear to be unreasonable from the viewpoint of communications requirements, the question arises as to whether present radars are capable of determining the orbit to the precision required to yield a drift rate of 1°/yr The answer is that basic tracker accuracies comparable to those of the deep space net[19] are sufficient for most 24-hr missions The reason for this is the admissibility of long smoothing times in these missions During the positioning phase, the allowed reaction time of the system approaches one day Following the positioning phase, the satellite should be reasonably stationary, and the reaction time of the system is extended to several days or perhaps weeks, depending on the specifications

To illustrate this, one can consider the matter of drift rate errors Figure 3 presents a curve of drift rate uncertainty (1 σ) vs smoothing time for a single station measuring range rate and angles The tracking geometry approximates that of Goldstone (member installation of the Deep Space Information Facility) in relation to a satellite centered over the western hemisphere The data sources were assumed to have random errors with 1 σ values of 1 fps and 0 06° A conservative observation rate of once per hour was assumed

From the figure we can see that a one-day tracking interval results in a 1 σ uncertainty in drift rate of 0 057°/day Assuming propulsion dispersions will add an equal statistical component, the final correction of the positioning phase will have a 1 σ error of 0 088°/day If a longitude specification of ±2° is used, it is seen that up to 8 days of tracking is permitted before the satellite exceeds this limit with a 3σ drift rate During this time, tracking uncertainties can be reduced to 0 001°/day or 0 01 fps, which is well within the 0 027 fps required to yield a drift rate of 1°/yr Therefore, the value of D_s to which the orbit control system must control the satellite was selected to be 1°/yr

C Satellite Configuration

The velocity accuracy required to obtain the specified value of D_s points out the need for a rather precise propulsive element in the satellite This may be obtained through the use of a cold-gas jet or other low-thrust devices On the other hand, biased-injection into a walking orbit may leave an orbital speed deficit anywhere from 200–2000 fps Satellite propellant requirements would be prohibitive unless a thrust element with at least a moderate specific impulse were used The positioning phase thus requires a hot-gas system For arbitrary longitude positioning, this system must be capable of at least two starts: one to establish the proper drift rate so that the satellite arrives on station at an apogee point; another to remove the existing drift rate upon arrival If properly exercised, the hot-gas system need only have a single nozzle, that is, a unilateral capability

These considerations, together with possible station-keeping requirements, lead to selecting a satellite propulsion system having high- and low-level thrusts with multiple start capabilities The exact means by which corrective impulses are vectored and metered are subject to design variations depending on the particular payload configuration, mission objectives, and instrumentation capabilities One possibility employs thrust-timing and body-fixed nozzles with active attitude control, and this method has been assumed for this system Timing is feasible in view of the satellite's low thrust-accelerations and the closed-loop action of the system Body-fixed nozzles with satellite attitude control is, of course, one of the most straightforward means of thrust-vectoring It is also a natural selection if the mission requires that the payload itself be controlled in attitude during free-flight Attitude references for such a system may be obtained by earth-horizon and sun sensors For equatorial missions, the principal thrust orientation is one that is locally horizontal and in the equatorial plane The horizon scanner will enable the pitch and roll channels to align the thrust axis in the local horizon plane Yaw alignment of the thrust axis into the equatorial plane is possible by first sensing the direction of the satellite-sun line projection in the local horizon plane and then developing a yaw rotation appropriate to the existing satellite-earth-sun geometry The success of this method of yaw control depends greatly on the earth-central angle between the satellite and the sun Control is optimum when this angle is 90° At other angles, foreshortening of the satellite-sun line projection occurs, and yaw referencing becomes less accurate The extreme condition of the sun directly overhead the satellite (noon condition) yields an indeterminate yaw reference unless a special noon-control mode is included [2, 3]

D Orbit Perturbations

The nature of the orbit control phase and its interface with longitude positioning requires consideration of perturbing influences on the orbit The effects of earth oblateness, lunar-solar gravitation, and ellipticity of the equator must be considered

1 *Earth oblateness*

The influence of earth oblateness on the elements of a near-circular orbit has been treated extensively in the literature [4–7] Of major concern is the effect of these perturbative forces on the longitude motion of the satellite The oblateness effect enters into calculation of the satellite's sidereal period, which is directly related to longitude drift If this effect is not accounted for in computing the sidereal period of the satellite, a longitude drift of 10°/yr is induced The following period computation may be used to account for the oblateness effect[8]:

$$T = \frac{2\pi}{\mu^{*1/2}}\left(\frac{r_0}{2 - (r_0 V_0{}^2/\mu^*)}\right)^{3/2} \quad (2)$$

$$\mu^* = \mu\left[1 + J\left(\frac{R}{r_0}\right)^2 + \frac{3}{7}D\left(\frac{R}{r_0}\right)^4\right] \quad (3)$$

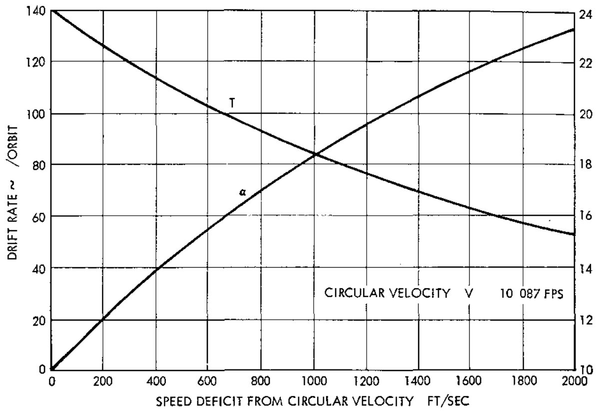

Fig 2 α, T vs apogee speed deficit

μ^* = equivalent μ, compensating for oblateness
J,D = oblateness constants
R = earth equatorial radius

The equivalent μ compensation reduces the longitude drift error to 0 0002°/day or 0 075°/yr

2 *Lunar-solar perturbations*

The effects of lunar-solar gravitation on earth-satellites have been analyzed by various researchers [9–11, 17] Periodic effects exist on all orbital elements in various combinations of harmonics of the earth, moon, and satellite orbital rates The total longitude deviation due to the periodic effects is bounded by 0 10° The secular drift is dependent on the satellite's initial phase with respect to the sun and moon and, thus, varies in magnitude However, it can be accounted for in the initial positioning phase In addition, lunar-solar gravitation induces a change in orbit inclination with the maximum change being 0 86°/yr

3 *Elliptic equator*

The effect of the elliptic equator on 24-hr satellites has been analyzed in Refs 12–16 From these references it is found that the elliptic equator causes a small change in the potential function, which varies with the longitude distance of the satellite from the elliptic equator's major axis In

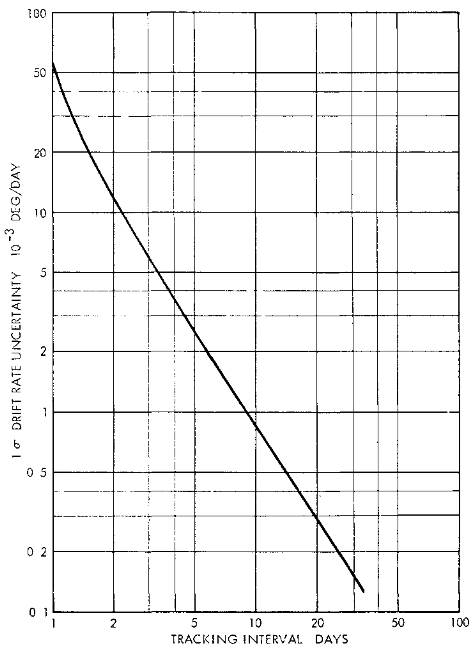

Fig 3 Drift rate uncertainty vs tracking time

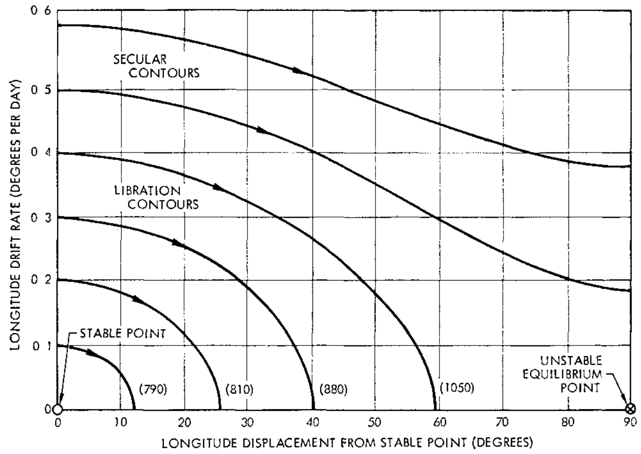

Fig 4 Apparent motion of 24-hr equatorial satellite due to elliptic equator

spite of the minuteness in potential variation, a near-synchronous satellite will experience long-period librations in longitude about the nearest minor axis longitude Figure 4 presents a phase-plane diagram of drift rate vs longitude position from a stable point The curves are symmetric about the coordinate axes shown, and the numbers within parentheses indicate the total libration period in days for complete transversal of a libration contour The period approaches a lower limit of about 785 days for motion in the immediate neighborhood of the stable point The secular contours show how the elliptic equator modifies the motion of a satellite with an initial drift rate outside of the librational range In the absence of equatorial ellipticity, all phase-plane trajectories would have been horizontal straight lines From the figure it can be seen that near-stationary orbits with initial drift rates less than $\frac{1}{2}$ deg per orbit (day) are substantially modified, whereas those with higher initial rates experience progressively less effect

The degree to which these perturbative effects must be considered and accounted for depends on the requirements placed on the system For long operating lifetimes and the specified value of D_s, these effects must be considered and corrected as will be discussed in the development of the orbit control logic to be used in the orbit control system

E Orbit Control Methods

The method of approach which was used in developing an orbit control logic for the 24-hr satellite was to consider the problem of longitude control from the standpoint of phase space, wherein the generalized velocity and distance coordinates are the satellite's drift rate and longitude relative to the desired final longitude To attain the final longitude, the satellite must remove the apogee speed deficit in such a manner that the desired position is obtained upon completion of control This is termed the positioning phase However, because of the librational movement induced by the elliptic equator, the satellite will in time drift away from the desired final longitude In order to maintain the satellite within specified longitude limits about the desired longitude, a limit cycle operation is thus required This is the role of station keeping

The two stable longitudes, however, are unique in that station keeping may not be required at these points This is because the elliptic equator provides natural limit cycling, and dispersed conditions about stable points tend to be localized A sufficient condition that precludes station keeping over stable longitudes is the capability of meeting specifications, assuming absence of the elliptic equator The natural keeping effect of the equator then assures motion within tolerance

1 *Hot-gas positioning phase*

The positioning method selected for this system utilizes multiple hot-gas corrections at apogee under a unilateral thrust constraint The initial drift rate is monotonically decreased in steps to avoid premature velocity and position overshoots, which can occur under dispersed conditions The method was originally developed by Gunckel[20] using what essentially amounts to a dynamic programming approach The hot-gas positioning phase control laws are summarized below:

a Mode H-1, normal operating mode

$$\Delta\alpha = \begin{cases} \dfrac{1}{1-K}\left((\alpha_{n-1} - \dfrac{D_n}{1+\delta}\right) & \text{if positive} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

b Mode H-2, transition mode Employed when $\epsilon_{\alpha n} < D_n \leq \epsilon_n,\ \eta(N_{\max},\delta,K)$

$$\Delta\alpha_n = \begin{cases} \alpha_{n-1} - D_n & \text{if positive} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

c Mode H-3, terminal mode Employed when $D_n < \epsilon_{\alpha n}$

$$\Delta\alpha_n = \begin{cases} \alpha_{n-1} - (D_n/N_c) & \text{if positive and } D_n > 0 \\ \alpha_{n-1} & \text{if positive and } D_n \leq 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where

$\Delta\alpha_n$	=	commanded drift rate reduction of nth firing
α_{n-1}	=	drift rate prior to nth firing
α_n	=	drift rate following nth firing
D_n	=	longitude deviation of satellite from the desired longitude, also termed distance-to-go at nth firing
K	=	3σ proportionality error due to propulsion
δ	=	$\delta(K)$, a logic parameter
ϵ_p	=	3σ additive propulsion error in α
ϵ_E	=	3σ error in estimating α from tracking
ϵ_D	=	3σ error in estimating D from tracking
ϵ_n	=	$(\epsilon_p{}^2 + \epsilon_E{}^2)^{1/2}$, total additive error in α_n
$\epsilon_{\alpha n}$	=	$(\epsilon_n{}^2 + K^2\Delta\alpha_n)^{1/2}$, total error in α_n
$N_{\max}, N_c$	=	logic parameters > 1

As noted, this control phase is divided into 3 operating modes

a Normal mode The normal mode control law is motivated by the fact that large initial drift rates require large drift rate reductions that, under the proportional errors of the system, yield large dispersions in turn In the ideal case, positioning of the satellite could be accomplished with two corrections by adjusting the drift rate to equal the remaining distance-to-go However, in the presence of propulsion proportional errors, large position and velocity overshoots could occur Therefore, to avoid position and velocity overshoots early in the hot-gas phase, the parameters K and δ are used K insures no position overshoot whereas δ insures no velocity overshoot

b Transition mode As the satellite approaches the desired longitude with a monotonic decreasing drift rate, the commanded drift rate reductions diminish in size Eventually, the proportional errors of the system will no longer be representative of the true statistics of the problem, and the additive errors must be taken into account Also, the constraint against position overshoots will no longer apply, since the drift rate will be very low When this occurs, the transition mode logic is used This logic is simply the ideal logic, i e , the drift rate is adjusted to equal the remaining distance to go

c Terminal mode The terminal mode logic commands the final hot-gas correction and is used only once It will normally be engaged only after the transition mode has been exercised Whenever the transition mode produces arrival on station or a position overshoot, the terminal mode removes all existing drift rate and then transfers control to the cold-gas system It is also used whenever the transition mode is likely to produce a velocity overshoot and the remaining distance to go is positive Therefore, a terminal hot-gas correction should be made and subsequent control transferred to the cold-gas system

2 *Cold-gas venier phase-stable longitude positioning*

As stated previously, no station keeping is required at a stable longitude if the system can initially control the drift rate to meet specifications in the absence of an elliptic equator Ignoring the added stability induced by the elliptic effect, the cold-gas logic summarized below is optimal with respect to time The notation of previous sections of the paper is used except that indicial subscripts have been deleted The reason for this is that the cold-gas phase need not decide on a correction after each orbit Instead, it should utilize variable tracking intervals so as to optimize the accuracy of each correction Should a correction be desired, it may be made at any point in the orbit, since the eccentricity will be extremely small Systems utilizing sun-sensors for satellite yaw referencing may take advantage of this freedom by scheduling cold-gas correction at optimum sun-aspect angles

a Mode C-1, initial correction To occur N_c days after hot-gas termination:

$$\Delta\alpha = \alpha - (D/N_L) \tag{7}$$

b Mode C-2, normal mode:

$$\Delta\alpha = \begin{cases} \alpha - (D/N_L) \text{ if } \epsilon_\alpha \leq (D_s/N_L) \\ 0 \text{ otherwise} \end{cases} \tag{8}$$

c Termination criterion:

$$\begin{aligned} |D| + \epsilon_D &\leq D_s \\ |\alpha - D/N_L| + \epsilon_E &\leq (D_s/N_L) \end{aligned} \tag{9}$$

Control termination criteria are included under the cold-gas control logic, since the hot-gas system will not usually be accurate enough to meet orbit specification These criteria are a simple test that insures that the remaining longitude distance to the desired longitude and the total longitude drift over the required system lifetime are within specification values before control is terminated The normal mode c g correction assumes that the remaining distance to go is within the specified longitude control limits about the stationary longitude This correction will then attempt to reduce the drift rate so that with high probability the adjusted state will be within a control dead zone

The initial cold-gas mode is used only once Its sole purpose is to act as an immediate venier following hot-gas termination so that the satellite's longitude is within tolerance during subsequent normal mode corrections Since tracking uncertainties will be dominant, this correction is scheduled N_c (>1) days after hot-gas termination in order to allow more smoothing than was possible during positioning This is done to assure that the correction can serve as an effective vernier

3 *Cold-gas phase-station keeping*

When the desired longitude does not coincide with a stable point, the cold-gas logic just discussed cannot be used Instead, a limit-cycle operation of the type shown in Fig 5 must be employed

As shown, the hot-gas positioning phase initially establishes the satellite at ϕ_s, the desired station relative to the nearest stable point The ensuing cold-gas phase then consists of an immediate vernier correction followed by periodic impulses that sustain a limit-cycle spanning the tolerance region The limiting phase path shown in the figure requires that the drift rate at $\phi = \phi_s - D$ be adjusted to $\dot{\phi}(\phi - D_s)$

The control logic associated with station keeping also consists of two modes

a Initial correction To occur N_c days after hot-gas termination:

$$\Delta\alpha = \alpha \tag{10}$$

b Normal keeping mode:

$$\Delta\alpha = \begin{cases} [1/(1+K)](\alpha - \alpha_{-D}) + \epsilon_n \text{ if } D = -D_s \\ 0 \text{ otherwise} \end{cases} \tag{11}$$

In these equations $\Delta\alpha$ still retains its definition as a commanded reduction in α Thus, for the example of Fig 5, the sign of $\Delta\alpha$ in (11) will be negative Note also that (11) assumes that the nearest stable point is west of the desired longitude Should it be east of ϕ, all negative signs in Eqs (10) and (11) become positive, and ϵ is replaced by $-\epsilon_n$

a Initial correction As in the case of stable longitude positioning, this correction serves as an immediate vernier to the hot-gas phase It differs only in that the complete drift rate is removed

b Normal keeping mode The derivation of (11) is straightforward It merely assures with 0 99 probability that the adjusted drift will not exceed α_{-D} in the presence of proportional and additive errors This assures motion within the limiting contour and, hence, satisfaction of specifications Of interest are the time and velocity requirements for the station-keeping action Figure 6 shows the velocity required per cycle for a longitude band of $D_s = \pm 5°$ about the desired stationary longitude Also shown is the approximate limit-cycle period From this we can see that the velocity required is modest and that long periods of time exist between corrections

F Control and Computations Center Operations

The control and computation center will direct and coordinate all operations following booster injection of the satellite Appointing a prospective orbit correction time, it will request the necessary tracking data, determine the orbit, and generate the satellite's ephemeris In addition, theoretical estimates on the accuracy of the derived information will be computed The derived orbit elements and ephemeris, including their accuracy estimates, are then incorporated in a logical operation to yield control decisions As the prospective correction time approaches, the control center will decide if a correction is actually desired If a correction is desired, it will derive the precise time at which thrusting should be initiated, the propulsion system to be used, the thrust direction required, and the required thrust duration The control center will then select the appropriate command unit, compute antenna pointing angles, and submit the instructions

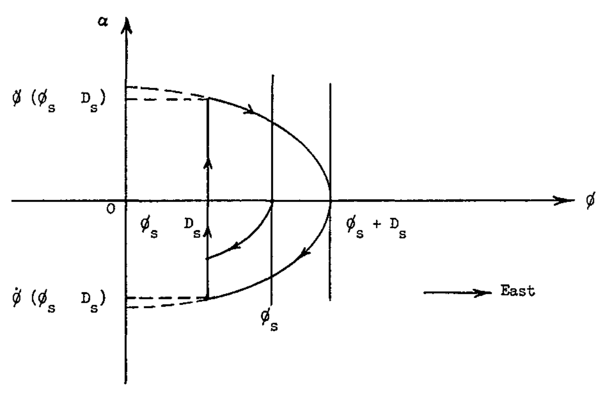

Fig 5 Station-keeping limit cycle operation

to the station Tracking stations will then be provided with instructions pertaining to the necessary amount of tracking data required for the next prospective correction If the control center originally decides against a correction, it will merely extend tracking and telemetry operations to the next prospective correction A correction will be postponed for one of two reasons; either because at the appointed time a correction is found to be unnecessary, or because greater benefit can be realized by using a longer tracking interval before commanding a correction The accuracy estimates on the derived information and a priori statistics of the satellite subsystem accuracy will enable the control center to employ the proper strategy The control center will cease to appoint prospective corrections after it decides that the satellite is in a satisfactory orbit

In order to accomplish the tasks just noted, the control center will require a high speed digital computer of the IBM 7090 class Stored in the computer will be a tracking program, including an orbital parameter calculations package, and the control logic previously presented Following is a description of how these programs would be incorporated into a computer at the control center to yield the required parameters for making an orbit correction

1 *Tracking program*

After the radar data are received, they are submitted to the tracking program for orbit determination The tracking program then performs an iterative least square curve fit on the observations to obtain a best estimate of the satellite's orbital parameters at an assigned epoch For the selected control logic, epoch is taken at "burnout" of all corrections For the initial tracking period, epoch is taken at initial orbit injection of the satellite

Having obtained a satisfactory set of parameter estimates, the tracking program then computes the equivalent information in terms of alternate coordinates, such as earth-centered rectangular coordinates and/or classical orbit elements It also estimates the covariance matrix for each set of coordinates by utilizing the set of partials computed Trajectory predictions and their associated accuracy estimates may then be obtained by integration and analytic computations Such tasks are officially regarded as tracking program tasks However, for the sake of a more coherent discussion, they are described under Orbital Computations

2 *Orbital computations*

The orbit control logic requires values of α, D, ϵ_E, and ϵ_D at the time of orbit correction These quantities are functions of the orbital parameters $R\delta\beta ArV$ and the covariance matrix of these parameters, where R is the right ascension, δ the declination, β the flight path angle, A the azimuth, r the radius, and V the velocity The $R\delta\beta ArV$ parameters and their covariance matrix are determined by the tracking

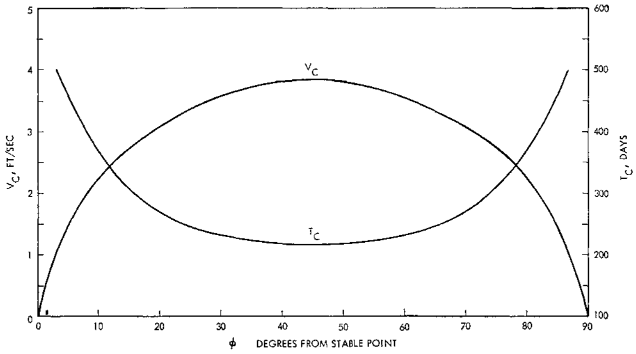

Fig 6 Limit cycle velocity requirements and period for $D_S = 5°$

program and input to the orbital computations package for translation into the control logic variables The orbital computation package makes the transformation of orbital data to logic data and feeds the logic data to the control logic

3 *Orbit control logic*

The orbit control logic to be used by the control center for determining whether or not a correction should be made is that discussed previously for the various control phases Figure 7 presents a skeletal diagram of how the hot- and cold-gas (stable longitude positioning) logics could be incorporated by the control center to derive the required orbit commands Following is a detailed discussion of the various phases noted on Fig 7

a Control termination-"orbit satisfactory" computation An important function of the orbit control logic is to determine when the system specifications on α and D have been met and then terminate orbit control This decision is made in the "orbit satisfactory" computation in relation to the hot- and cold-gas phase control logics Since this computation is carried out at each desired correction time, the orbit satisfactory box belongs to neither control phase but to the total control logic

According to Eq (9), orbit control may be terminated when a control dead-zone is entered In order to be certain with 3 σ probability that the satellite is in the control dead-zone and control may be terminated, the following control termination criteria are used:

$$
\begin{gathered}
|D_n| + e_D \leq D_{\max} \\
|\alpha_{n-1} - (D_n/N_L)| + \epsilon_E \leq \alpha_{\max}
\end{gathered}
\tag{12}
$$

If these criteria are satisfied, orbit control is terminated Orbit control is also terminated upon allowable fuel depletion in the cold-gas control phase

b Hot-gas phase logic Values of α, D, ϵ_D, and ϵ_E computed in the orbital computation package at the desired correction time enter the phase control box common to both the hot- and cold-gas logic sequences The hot-gas phase logic is entered when the value of α is greater than the specified constant K_{11}, and δ_1 is equal to one The value of K_{11} is based on the control capability of the cold-gas system The logic is designed for optimum operation with a zero value of K_{11} The δ_1 flag is initially set equal to one and is changed to zero only when a terminal mode correction is made or when a signal from the command translation box indicates that fuel depletion has occurred Once the $\alpha > K_{11}$ and $\delta_1 = 1$ criteria are satisfied and the hot-gas phase is entered, the problem is to determine which hot-gas control mode should be employed to calculate $\Delta\alpha$ This decision is made by the mode control logic

(i) Hot-gas phase mode control The first decision in the mode control determines whether or not D is within the control capability of the cold-gas system If $D < K_{12}$, the terminal mode is entered to make the final hot-gas correction and to transfer control to the cold-gas system as previously discussed If $D > K_{12}$, the positioning capability of the hot-gas phase is still required and, hence, the terminal mode is not entered The optimum value of K_{12}, like K_{11}, is dependent on the cold-gas system control capability, and a value of zero is to be used with the presented logic

If the terminal mode is not entered, the mode control must decide whether the normal or transition mode should make the orbit correction This decision is based on the switching criterion $K_{13}\,\epsilon_n{}^2$, where

$$
\begin{aligned}
K_{13} &= \eta^2 \\
\eta &= \frac{(1+\delta)(N_{\max}+1)[\delta(1+K)+2K]}{[\delta(1-K)-2K]N_{\max}-2K-\delta^2(1-K)} \\
\epsilon_n{}^2 &= \epsilon_E{}^2 + \epsilon_p{}^2
\end{aligned}
\tag{13}
$$

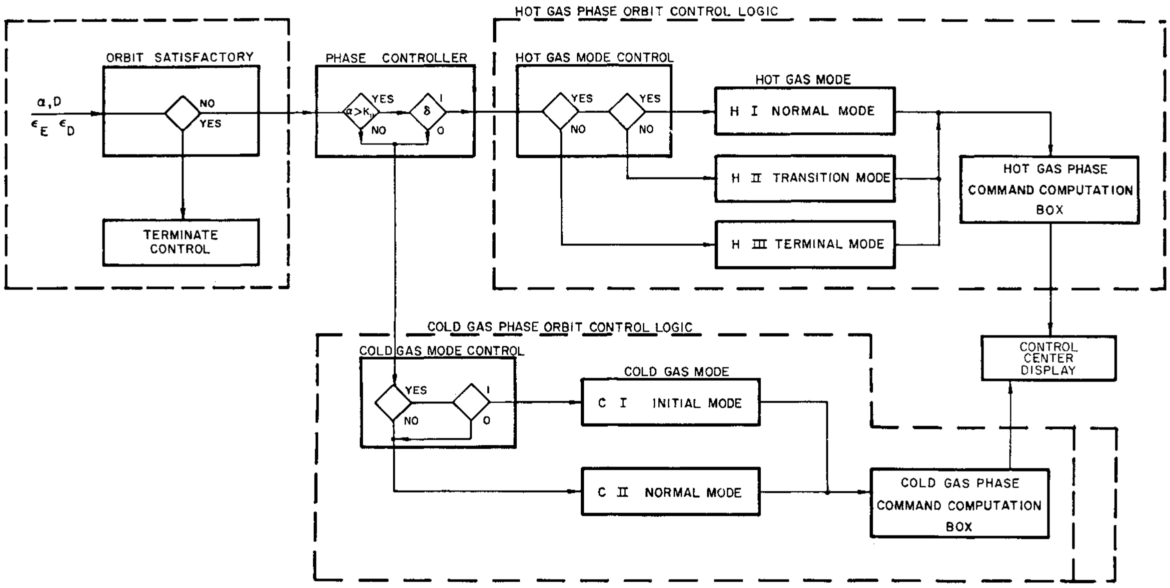

Fig 7 Skeletal diagram of orbit control logic

If $D^2 > K_{13}\epsilon_n^2$, the switching criterion is not satisfied, and the normal mode is used If $D^2 < K_{13}\epsilon_n^2$, the transition mode is used

Once the mode control box has determined which hot-gas mode is to be used, that mode is entered to calculate the value of $\Delta\alpha$ The logic employed in each of the modes is the logic previously described for each mode

(ii) Command computation The value of $\Delta\alpha$ computed in any of the hot-gas modes is sent to the command translation box for transformation into a time of burning (thrusting) command to be used for the actual orbit correction The transformation of $\Delta\alpha$ into velocity-to-be-gained is given by Eq (14):

$$\Delta V = V \times \left\{\left(1 + \frac{1}{V^2}\left(\frac{2\pi\mu^*}{T}\right)^{2/3}\left[1 - \left(\frac{1}{1 + (\Delta\alpha/\omega_S T)}\right)^{2/3}\right)^{1/2} - 1\right\} \tag{14}$$

where

V = velocity of satellite at time of firing
μ^* = equivalent μ of earth, compensating for oblateness[8]
T = sidereal period of the satellite
ω = sidereal rate of the earth

The time of burning is obtained from the well-known rocket equation:

$$t_b = (W_i/\dot{w})[1 - e(-\Delta V/gI_{\text{p}})] \tag{15}$$

where ΔV is given by Eq (14)

Before this command is made available to the control center, together with an indicator that the hot-gas propulsion is to be used for the next orbit correction, the required burning time is compared with the quantum time of the propulsion system If $t_b < t_{qh}$, the orbit control procedure is recycled for one more orbit If $t_b > t_{qh}$, a summing box is entered and t_b is added to the sum of all previous commanded thrust durations This sum

$$\sum_{i=1}^{n} t_{bi}$$

is compared with the total allowable burning time T_h, which is a function of the initially loaded propellant When

$$\sum^{n} t < T_h$$

the value of t_{bn} computed from $\Delta\alpha$ is sent directly to the control center If

$$\sum_{i=1}^{n} t_{bi} > T_h$$

fuel depletion will occur before the total commanded burning time has elapsed The value of t_b is still sent to the control center in order to make as much of the commanded correction as possible δ_1 is then set equal to zero in the phase control box, which automatically switches control to the cold-gas phase at the next correction time This step ensures continuous orbit control

c Cold-gas phase logic-stable longitude positioning The cold-gas phase logic to be used at the control center can be incorporated as shown in Fig 7 The required logic inputs from the orbital computations box enter the phase control box, which determines the proper phase to be used to compute the orbit correction The cold-gas phase is entered when $\alpha < K_{11}$, or when δ_1 is set equal to zero by the hot-gas logic thus indicating termination of hot-gas control Once the cold-gas phase is entered, the cold-gas mode control determines which mode should be employed to calculate $\Delta\alpha$

(i) Cold-gas phase mode control The mode control first determines whether or not the hot-gas phase has terminated by checking the value of δ_1 If $\delta_1 = 0$, the hot-gas phase has terminated and the mode control now determines if the initial cold-gas mode has been used by checking the value of δ_2 If $\delta_2 = 1$, the initial mode has not been used previously and, therefore, is entered for calculation of $\Delta\alpha$ If $\delta_2 = 0$, the initial mode has been used and control is therefore switched to the normal mode

The forementioned discussion has assumed that the cold-gas phase was entered after completion of the hot-gas phase It should be noted that the cold-gas phase can also be entered before δ_1 is set equal to zero in the hot-gas phase In this case $\alpha < K_{11}$, $\delta_1 = 1$, and the normal mode of the cold-gas phase is entered This is to safeguard against the possible case where α is small or negative but D is still large The cold-gas phase would then increase the value of α to reduce the time to reach the final desired longitude with a high probability that $\alpha > K_{11}$ at time of the next orbit correction If this is the case, the hot-gas phase is now used since $\delta_1 = 1$

(ii) Mode C-I—initial mode The logic shown in Fig 7 for the initial mode calculation of $\Delta\alpha$ is that presented in Eq (7) This calculation is made two orbits after the last commanded hot-gas correction The initial mode logic must also transfer control to the normal mode after making one correction, and this is done by setting $\delta_2 = 0$ in the mode control box after computing $\Delta\alpha$ All corrections thereafter are then made by the normal mode

(iii) Mode C-II—normal mode The normal mode control logic to be used is given in Eq (8) This logic is incorporated as shown in Fig 7 The tracking interval between the initial mode cold-gas correction and the first normal mode correction is that required to reduce $\epsilon\alpha < \alpha_{max}$ If the orbit is not satisfactory at this time and $\epsilon\alpha$ is still greater than α_{max}, then a one orbit tracking interval extension is used until the orbit becomes satisfactory or a correction is made If $\epsilon_\alpha < \alpha_{max}$ and the orbit is not satisfactory, then the computed value of $\Delta\alpha$ is made the commanded value and a correction is made Orbit trimming control is then terminated, and this phase of orbit control is assumed to be complete The orbit-keeping phase of the orbit control logic is then entered for determination of all future corrections

(iv) Command computation The command computation for the cold-gas phase is the same as for the hot-gas phase, only the proper cold-gas propulsion system nozzle is indicated to the control center In addition, if the sum of t_{b_n} indicates allowable propellant depletion will occur during a correction, orbit control is terminated after a command is sent to the satellite to make as much of the commanded correction as possible

d Cold-gas phase logic-orbit keeping When the satellite is to be positioned at a longitude other than a stable longitude, orbit control over the entire satellite's lifetime is required The orbit control logic, which should be employed at the control center to determine the required orbital corrections, is that presented previously This logic can be incorporated at the control center in a manner similar to that shown for the stable longitude positioning logic

III Orbit Control System Simulation and Study Results

The orbit control system just discussed has been simulated on an IBM 7090 digital computer in order to realistically test the capability of the control logic to position and control the satellite in a satisfactory manner The system simulation is built around a streamlined version of a general tracking program, and with the incorporation of ground station and powered flight simulation, as well as orbit control logic and computation packages, all essential features of the operational orbit control system are simulated in detail This allows comprehensive evaluation of each correction and decision as well as of over-all performance Printouts compare real conditions with system-apparent conditions throughout the simulation and record the entire sequence of logic decisions by diagram prints of the orbit control logic flow

Although the simulation is detailed and comprehensive, it none-the-less features high speed operation because of a minimum use of tape storage Except for the use of an ephemeris tape, all operations are entirely in IBM 7090 core storage Complete simulation of the biased injection ascent mission, for instance, is possible in 15 min

In order to obtain a realistic evaluation of the capability of the orbit control system, a Monte Carlo study was used In the Monte Carlo study, a nominal run was first obtained assuming no error in radar observations, propulsion system operation, or attitude control A series of runs was then obtained with noise errors (generated from a random number generator) added to the radar observations, commanded burning time, and satellite attitude at initiation of a commanded correction The generated noise was an uncorrelated, Gaussian noise with zero mean and standard deviation of one and with the standard deviations of the affected subsystem parameters used as weighting factors on the generated noise Each run used a different sequence of random numbers so that different errors were obtained in each run In this manner, the control system capability was tested under many possible error combinations In this study, a sample of 25 Monte Carlo runs was obtained

Only the hot-gas positioning and cold-gas (stable longitude positioning) orbit control logics were simulated for this study Inclusion of the elliptic equator and simulation of the required cold-gas phase orbit-keeping logic was considered and rejected because of the large amount of computer time required to thoroughly test the logic and the present uncertainty in the location of the stable longitudes [21–24] In all runs the biased injection ascent trajectory and a final desired longitude of approximately 100°W were assumed The Monte Carlo runs are representative of an orbit control system utilizing a Hawaii-Southern California tracking system having the accuracies previously mentioned Initial satellite injection errors were not included in the Monte Carlo runs

The specification values used were a satellite lifetime of 5 years and an allowable residual drift rate of 1°/orbit:

$\alpha_{max} = \pm 0\ 0027°/\text{orbit} = 0\ 67$ sec in satellite orbital period

$D_{max} = \pm 5°$

$N_L = 1825$ orbits (5 yr)

A Results of Nominal Run

The nominal run, as with all runs simulated, used a simulated injection time of January 1, 1963, at 86 6°E longitude A total of 5 hot-gas corrections was commanded over a simulated interval of $4\frac{1}{2}$ days The cold-gas phase, however, required only one correction This was the initial mode correction, as scheduled Following this correction, 14 days of tracking were used, after which the control logic decided the orbit to be satisfactory It then terminated control, with the total simulated control time being $20\frac{1}{2}$ days The residual drift rate and longitude error at termination were $\alpha = 0\ 0002°$/day, longitude error $= 0\ 0002°$ west $e = 0\ 0000863$, and $i = 0.0656°$

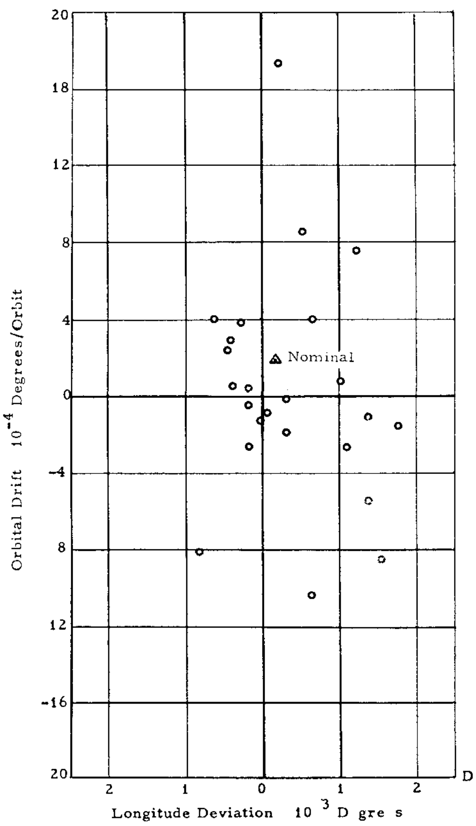

Fig 8 Scatter diagram of α and D

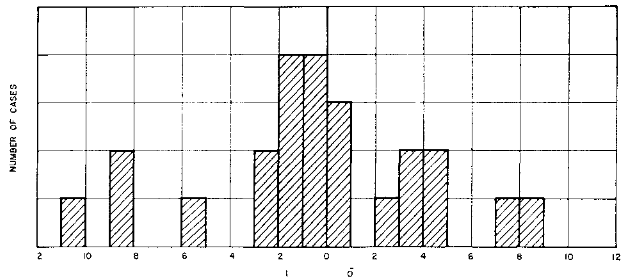

Fig 9 Histogram of orbital drift rate

B Results of Monte Carlo Study

Twenty-five Monte Carlo runs were then made with all runs being successful in achieving the required precision Two of the runs required only a single cold-gas correction, whereas the remaining runs utilized the maximum scheduled number of two Like the nominal, all runs used 5 hot-gas corrections and accomplished the mission in approximately 21 days

Figure 8 presents a scatter diagram of the residual drift rates and longitude deviations As can be seen, all points lie well within the allowable region of control errors Figure 9 shows the distribution of residual drift rates in histogramic form Even with as few as 25 samples, the Gaussian tendency is obvious One would suspect a priori, using the Central Limit Theorem, that α and D should have a bivariate Gaussian distribution Figure 9 and the scatter diagram of Fig 8 suggest this, indicating a suitable scatter in spite of the small number of runs

The sample mean and standard deviation of α and D are:

$$\bar{\alpha} = 0\ 43 \times 10^{-4} \text{ deg/oibit} \qquad \bar{D} = 0\ 37 \times 10^{-3} \text{ deg}$$
$$\bar{\sigma}_\alpha = 5\ 50 \times 10^{-4} \text{ deg/orbit} \qquad \bar{\sigma}_D = 0\ 77 \times 10^{-3} \text{ deg}$$

The largest values of eccentricity and inclination obtained at the end of the control period were $1\ 6 \times 10^{-4}$ and 0 13 deg, respectively

C Conclusions

The conclusions one draws from these results is that the hypothesized system is capable of satisfactorily positioning and controlling the satellite in the presence of likely radar, propulsion, and attitude errors and that the derived control logic could be used in real time with a similar type system and provide a satisfactory stationary orbit

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